

6.1

SOON YOU WILL DETERMINE THE RIGHT TRIANGLE CONNECTION

The Pythagorean Theorem

Learning Goals

In this lesson, you will:

- ▶ Use mathematical properties to discover the Pythagorean Theorem.
- ▶ Solve problems involving right triangles.

Key Terms

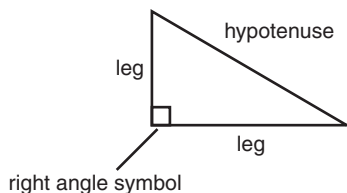
- | | |
|------------------------|-----------------------|
| ▶ right triangle | ▶ Pythagorean Theorem |
| ▶ right angle | ▶ theorem |
| ▶ leg | ▶ postulate |
| ▶ hypotenuse | ▶ proof |
| ▶ diagonal of a square | |

What do firefighters and roofers have in common? If you said they both use ladders, you would be correct! Many people who use ladders as part of their job must also take a class in ladder safety. What type of safety tips would you recommend? Do you think the angle of the ladder is important to safety?

Problem 1 Identifying the Sides of Right Triangles



A **right triangle** is a triangle with a right angle. A **right angle** has a measure of 90° and is indicated by a square drawn at the corner formed by the angle. A **leg** of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle. The **hypotenuse** of a right triangle is the longest side. The hypotenuse is opposite the right angle.



1. The side lengths of right triangles are given. Determine which length represents the hypotenuse.

a. 5, 12, 13

b. 1, 1, $\sqrt{2}$

c. 2.4, 5.1, 4.5

d. 75, 21, 72

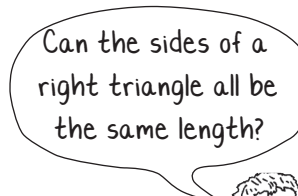
e. 15, 39, 36

f. 7, 24, 25

2. How did you decide which length represented the hypotenuse?



6



Problem 2 Exploring Right Triangles



In this problem, you will explore three different right triangles. You will draw squares on each side of the triangles and then answer questions about the completed figures.

A **diagonal of a square** is a line segment connecting opposite vertices of the square. Let's explore the side lengths of more right triangles.

1. An isosceles right triangle is drawn on the grid shown on page 317.
 - a. A square on the hypotenuse has been drawn for you. Use a straightedge to draw squares on the other two sides of the triangle. Then use different colored pencils to shade each small square.
 - b. Draw two diagonals in each of the two smaller squares.
 - c. Cut out the two smaller squares along the legs. Then, cut those squares into fourths along the diagonals you drew.
 - d. Redraw the squares on the figure in the graphic organizer on page 327. Shade the smaller squares again.
 - e. Arrange the pieces you cut out to fit inside the larger square on the graphic organizer. Then, tape the triangles on top of the larger square.



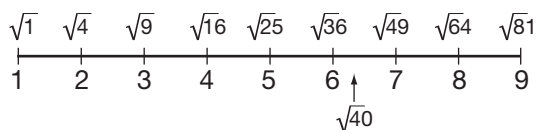
Answer these questions in the graphic organizer.

- f. What do you notice?
- g. Write a sentence that describes the relationship among the areas of the squares.
- h. Determine the length of the hypotenuse of the right triangle. Justify your solution.

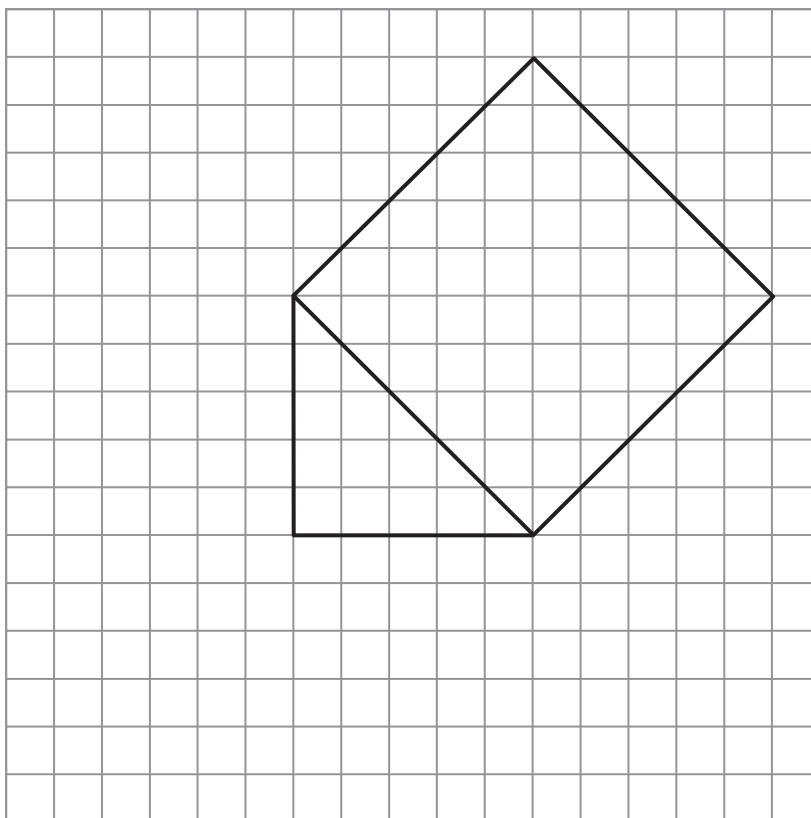
Remember,
 $A = s^2$ so,
 $\sqrt{A} = s$.



Remember that you can estimate the value of a square root by using the square roots of perfect squares.



The square root of 40 is between $\sqrt{36}$ and $\sqrt{49}$, or between 6 and 7. $\sqrt{40} \approx 6.3$.





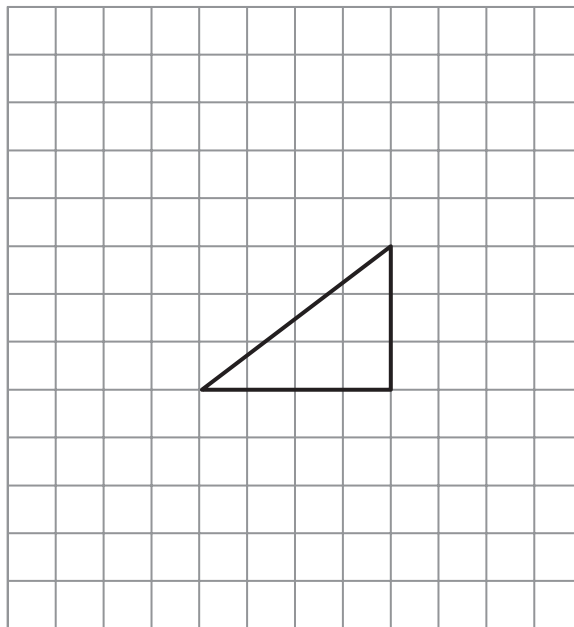
2. A right triangle is shown on page 321 with one leg 4 units in length and the other leg 3 units in length.
 - a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.
 - b. Cut out the two smaller squares along the legs.
 - c. Cut the two squares into strips that are either 4 units by 1 unit or 3 units by 1 unit.
 - d. Redraw the squares on the figure in the graphic organizer on page 328. Shade the smaller squares again.
 - e. Arrange the strips and squares you cut out on top of the square along the hypotenuse on the graphic organizer. You may need to make additional cuts to the strips to create individual squares that are 1 unit by 1 unit. Then, tape the strips on top of the square you drew on the hypotenuse.

Answer these questions in the graphic organizer.

- f. What do you notice?
- g. Write a sentence that describes the relationship among the areas of the squares.
- h. Determine the length of the hypotenuse. Justify your solution.

Remember, the length of the side of a square is the square root of its area.





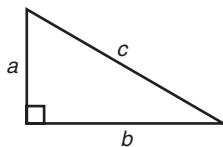
3. A right triangle is shown on page 325 with one leg 2 units in length and the other leg 4 units in length.
- Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.
 - Cut out the two smaller squares.
 - Draw four congruent right triangles on the square with side lengths of 4 units. Then, cut out the four congruent right triangles you drew.
 - Redraw the squares on the figure in the graphic organizer on page 329. Shade the smaller squares again.
 - Arrange and tape the small square and the 4 congruent triangles you cut out over the square that has one of its sides as the hypotenuse.

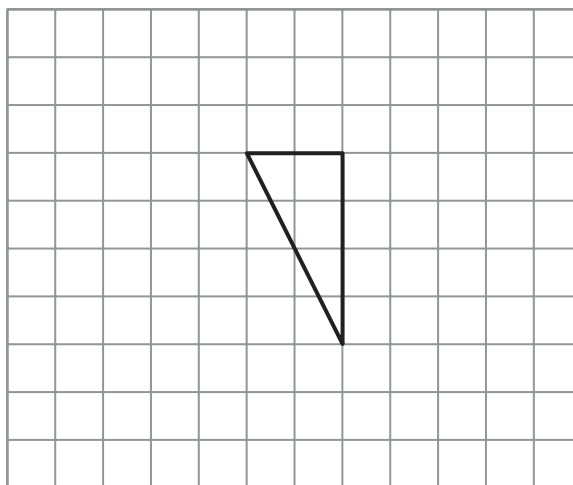
Answer these questions in the graphic organizer.

- What do you notice?
 - Write a sentence that describes the relationship among the areas of the squares.
 - Determine the length of the hypotenuse. Justify your solution.
4. Compare the sentences you wrote for part (f) in Questions 1, 2, and 3. What do you notice?



5. Write an equation that represents the relationship among the areas of the squares. Assume that the length of one leg of the right triangle is “ a ,” the length of the other leg of the right triangle is “ b ,” and the length of the hypotenuse is “ c .”

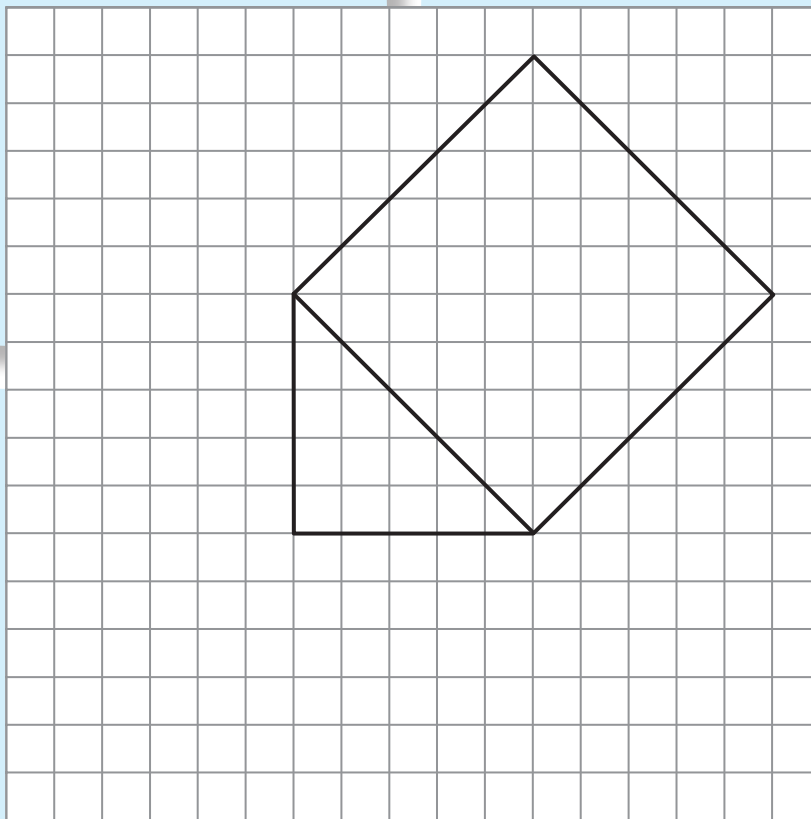




RIGHT TRIANGLE: BOTH LEGS WITH LENGTH OF 5 UNITS

WHAT DO YOU NOTICE?

**DESCRIBE THE RELATIONSHIP
AMONG THE AREAS OF
THE SQUARES.**

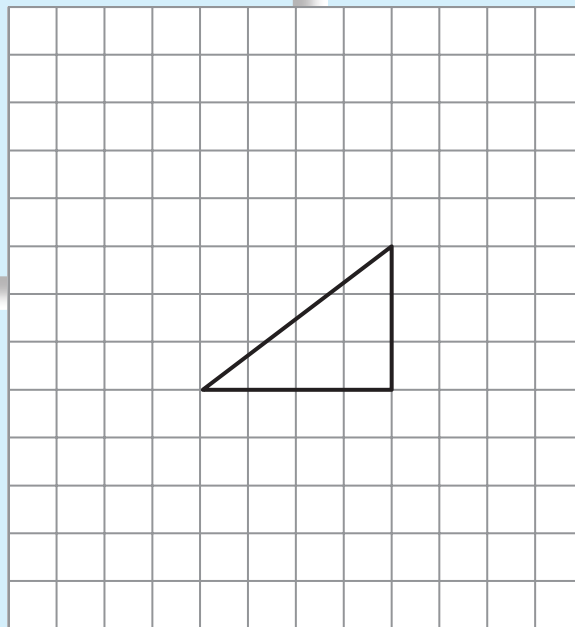


DETERMINE THE LENGTH OF THE HYPOTENUSE.

**RIGHT TRIANGLE: ONE LEG WITH LENGTH OF 4 UNITS
AND THE OTHER LEG WITH LENGTH OF 3 UNITS**

WHAT DO YOU NOTICE?

**DESCRIBE THE RELATIONSHIP
AMONG THE AREAS OF
THE SQUARES.**

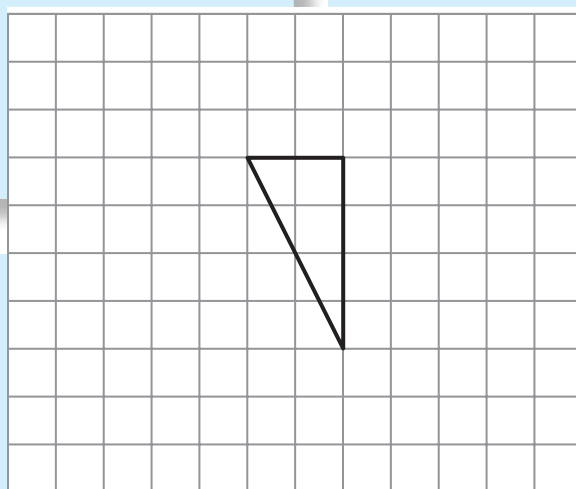


DETERMINE THE LENGTH OF THE HYPOTENUSE

**RIGHT TRIANGLE: ONE LEG WITH LENGTH OF 2 UNITS
AND THE OTHER LEG WITH LENGTH OF 4 UNITS**

WHAT DO YOU NOTICE?

**DESCRIBE THE RELATIONSHIP
AMONG THE AREAS OF
THE SQUARES.**



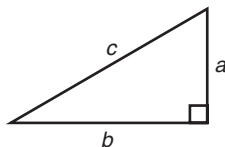
DETERMINE THE LENGTH OF THE HYPOTENUSE.

Problem 3 Special Relationships



The special relationship that exists between the squares of the lengths of the sides of a right triangle is known as the *Pythagorean Theorem*. The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

The **Pythagorean Theorem** states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.



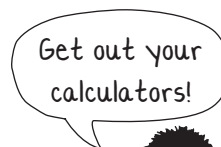
A **theorem** is a mathematical statement that can be proven using definitions, *postulates*, and other theorems. A **postulate** is a mathematical statement that cannot be proved but is considered true. The Pythagorean Theorem is one of the earliest known to ancient civilization and one of the most famous. This theorem was named after Pythagoras (580 to 496 B.C.), a Greek mathematician and philosopher who was the first to *prove* the theorem. A **proof** is a series of steps used to prove the validity of a theorem. While it is called the Pythagorean Theorem, the mathematical knowledge was used by the Babylonians 1000 years before Pythagoras. Many proofs followed that of Pythagoras, including ones proved by Euclid, Socrates, and even the twentieth President of the United States, President James A. Garfield.



1. Use the Pythagorean Theorem to determine the length of the hypotenuse:

- a. in Problem 2, Question 1.

- b. in Problem 2, Question 3.



Problem 4 Maintaining School Grounds

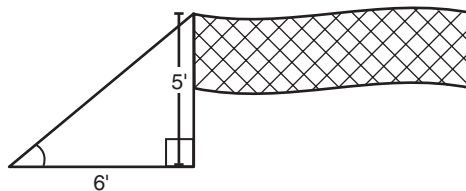
Mitch maintains the Magnolia Middle School campus. Use the Pythagorean Theorem to help Mitch with some of his jobs.



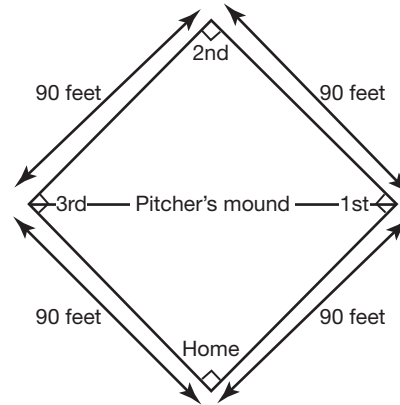
1. Mitch needs to wash the windows on the second floor of a building. He knows the windows are 12 feet above the ground. Because of dense shrubbery, he has to put the base of the ladder 5 feet from the building. What ladder length does he need?



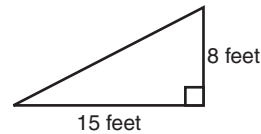
2. The gym teacher, Ms. Fisher, asked Mitch to put up the badminton net. Ms. Fisher said that the top of the net must be 5 feet above the ground. She knows that Mitch will need to put stakes in the ground for rope supports. She asked that the stakes be placed 6 feet from the base of the poles. Mitch has two pieces of rope, one that is 7 feet long and a second that is 8 feet long. Will these two pieces of rope be enough to secure the badminton poles? Explain your reasoning.



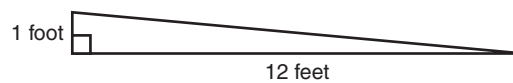
3. Mitch stopped by the baseball field to watch the team practice. The first baseman caught a line drive right on the base. He touched first base for one out and quickly threw the ball to third base to get another out. How far did he throw the ball?



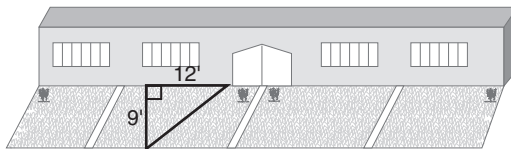
4. The skate ramp on the playground of a neighboring park is going to be replaced. Mitch needs to determine how long the ramp is to get estimates on the cost of a new skate ramp. He knows the measurements shown in the figure. How long is the existing skate ramp?



5. A wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the wheelchair ramp be?



6. The eighth-grade math class keeps a flower garden in the front of the building. The garden is in the shape of a right triangle, and its dimensions are shown. The class wants to install a 3-foot-high picket fence around the garden to keep students from stepping onto the flowers. The picket fence they need costs \$5 a linear foot. How much will the fence cost? Do not calculate sales tax. Show your work and justify your solution.

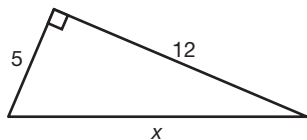


Problem 5 Solving for the Unknown Side

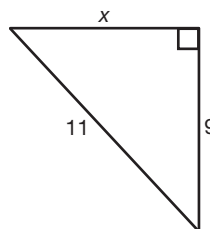


1. Write an equation to determine each unknown length. Then, solve the equation. Make sure your answer is simplified.

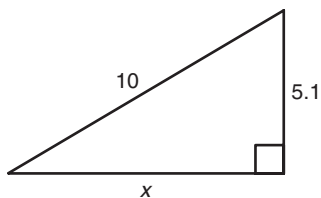
a.



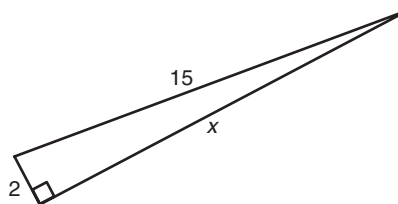
b.



c.



d.



6



Be prepared to share your solutions and methods.

6.2

CAN THAT BE RIGHT?

The Converse of the
Pythagorean Theorem

Learning Goal

In this lesson, you will:

- ▶ Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine unknown side lengths in right triangles.

Key Terms

- ▶ converse
- ▶ Converse of the Pythagorean Theorem
- ▶ Pythagorean triple

“Mind your p ’s and q ’s!” This statement usually refers to reminding a person to watch their manners. While the definition is easy to understand, the origin of this saying is not clear. Some people think that it comes from a similar reminder for people to remember their “please and thank-yous” where the “ q ’s” rhymes with “yous.” Others believe that it was a reminder to young children not to mix up p ’s and q ’s when writing because both letters look very similar.

However, maybe the origin of this saying comes from math. When working with theorems (as you did in the last lesson), mathematicians encounter if-then statements. Often, if-then statements are defined as “if p , then q ,” with the p representing an assumption and the q representing the outcome of the assumption. So, just maybe math played a role in this saying.

Problem 1 The Converse



The Pythagorean Theorem can be used to solve many problems involving right triangles, squares, and rectangles. The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse length equals the sum of the squares of the leg lengths. In other words, if you have a right triangle with a hypotenuse of length c and legs of lengths a and b , then $a^2 + b^2 = c^2$.

The **converse** of a theorem is created when the if-then parts of that theorem are exchanged.

The **Converse of the Pythagorean Theorem** states that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

If the lengths of the sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.



1. Determine whether the triangle with the given side lengths is a right triangle.

a. 9, 12, 15

b. 24, 45, 51

c. 25, 16, 9

d. 8, 8, 11





You may have noticed that each of the right triangles in Question 1 had side lengths that were integers. Any set of three positive integers a , b , and c that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple**. For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.



2. Complete the table to identify more Pythagorean triples.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple	3	4	5	$9 + 16 = 25$
Multiply by 2				
Multiply by 3				
Multiply by 5				

What if I multiplied 3, 4, and 5 each by a decimal like 2.2? Would those side lengths form a right triangle?

3. Determine a new Pythagorean triple not used in Question 2, and complete the table.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple				
Multiply by 2				
Multiply by 3				
Multiply by 5				

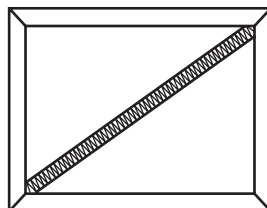


4. Record other Pythagorean triples that your classmates determined.

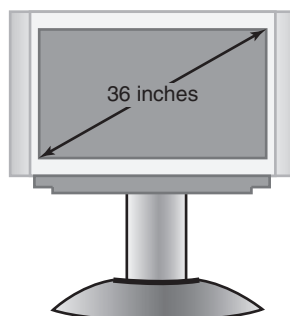
Problem 2 Solving Problems

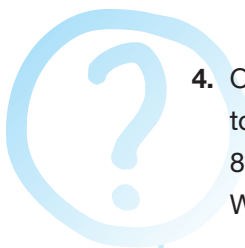


1. A carpenter attaches a brace to a rectangular-shaped picture frame. If the dimensions of the picture frame are 30 inches by 40 inches, what is the length of the brace?



2. Bill is building a rectangular deck that will be 8 feet wide and 15 feet long. Tyrone is helping Bill with the deck. Tyrone has two boards, one that is 8 feet long and one that is 7 feet long. He puts the two boards together, end to end, and lays them on the diagonal of the deck area, where they just fit. What should he tell Bill?
3. A television is identified by the diagonal measurement of the screen. A television has a 36-inch screen whose height is 22 inches. What is the length of the television screen? Round your answer to the nearest inch.



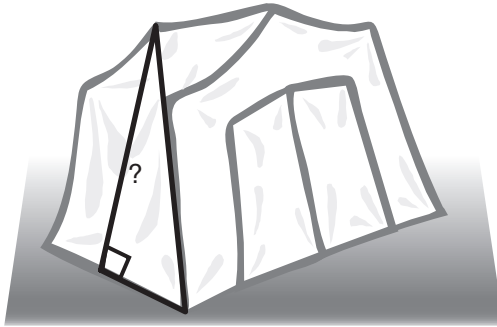


4. Orville and Jerri want to put a custom-made, round table in their dining room. The table top is made of glass with a diameter of 85 inches. The front door is 36 inches wide and 80 inches tall. Orville thinks the table top will fit through the door, but Jerri does not. Who is correct and why?

5. Sherie makes a canvas frame for a painting using stretcher bars. The rectangular painting will be 12 inches long and 9 inches wide. How can she use a ruler to make sure that the corners of the frame will be right angles?

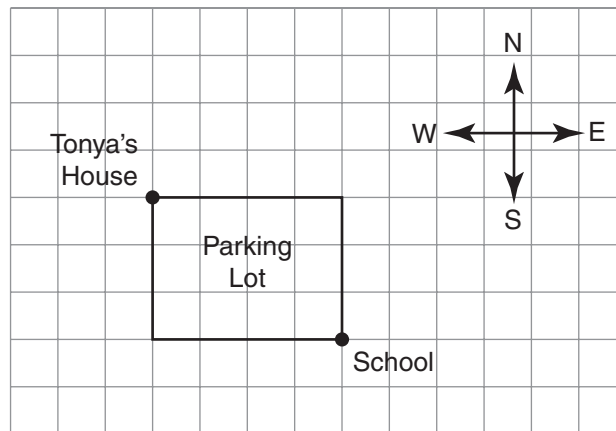
6. A 10-foot ladder is placed 4 feet from the edge of a building. How far up the building does the ladder reach? Round your answer to the nearest tenth of a foot.

7. Chris has a tent that is 64 inches wide with a slant length of 68 inches on each side. What is the height of the center pole needed to prop up the tent?



8. A ship left shore and sailed 240 kilometers east, turned due north, then sailed another 70 kilometers. How many kilometers is the ship from shore by the most direct path?

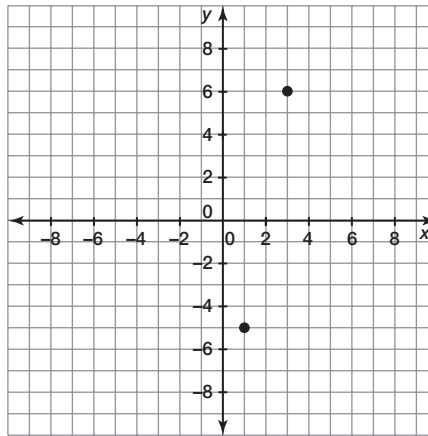
9. Tonya walks to school every day. She must travel 4 blocks east and 3 blocks south around a parking lot. Upon arriving at school, she realizes that she forgot her math homework. In a panic, she decides to run back home to get her homework by taking a shortcut through the parking lot.



- a. Describe how many blocks long Tonya's shortcut is.
- b. How many fewer blocks did Tonya walk by taking the shortcut?
10. Danielle walks 88 feet due east to the library from her house. From the library, she walks 187 feet northwest to the corner store. Finally, she walks 57 feet from the corner store back home. Does she live directly south of the corner store? Justify your answer.

11. What is the diagonal length of a square that has a side length of 10 cm?

12. Calculate the length of the segment that connects the points $(1, -5)$ and $(3, 6)$.



- a. Write your answer as a radical.
- b. Write your answer as a decimal rounded to the nearest hundredth.



Be prepared to share your solutions and methods.

6.3

PYTHAGORAS TO THE RESCUE

Solving for Unknown Lengths

Learning Goal

In this lesson, you will:

- ▶ Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine the unknown side lengths in right triangles.

There's a very famous mathematical scene in the movie *The Wizard of Oz*. At the end, when the wizard helps the scarecrow realize that he has had a brain all along, the scarecrow says this:

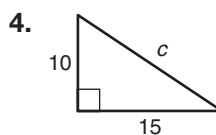
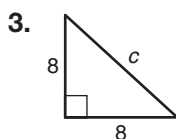
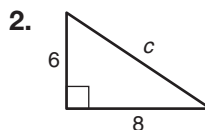
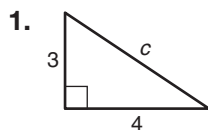
"The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side. Oh joy! Rapture! I've got a brain! How can I ever thank you enough?"

What did the scarecrow get wrong?

Problem 1 Determining the Length of the Hypotenuse

In this lesson, you will investigate solving for different side lengths of right triangles and using the converse of the Pythagorean Theorem.

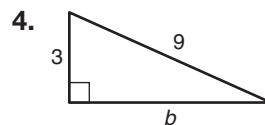
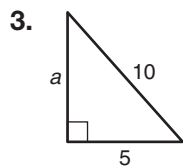
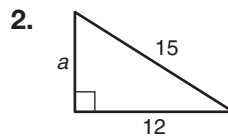
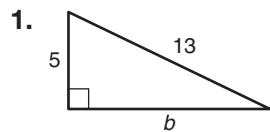
Determine the length of the hypotenuse of each triangle. Round your answer to the nearest tenth, if necessary.



Problem 2 Determining the Length of a Leg



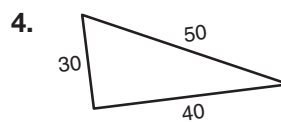
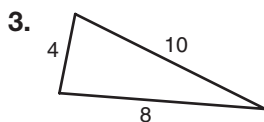
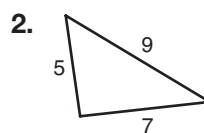
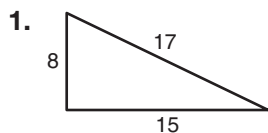
Determine the unknown leg length. Round your answer to the nearest tenth, if necessary.



Problem 3 Determining the Right Triangle



Use the converse of the Pythagorean Theorem to determine whether each triangle is a right triangle. Explain your answer.



Problem 4 Determining the Unknown Length

Use the Pythagorean Theorem to calculate each unknown length. Round your answer to the nearest tenth, if necessary.



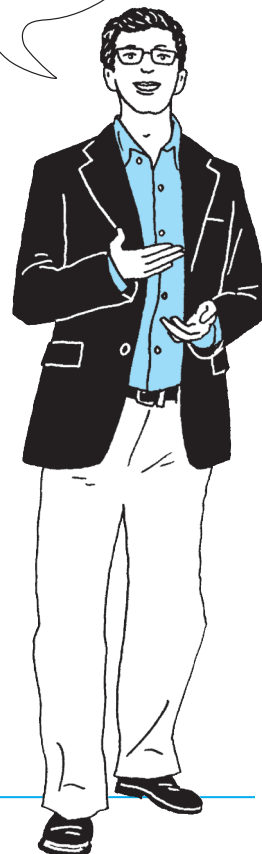
1. Chandra has a ladder that is 20 feet long. If the top of the ladder reaches 16 feet up the side of a building, how far from the building is the base of the ladder?

2. A scaffold has a diagonal support beam to strengthen it. If the scaffold is 12 feet high and 5 feet wide, how long must the support beam be?

3. The length of the hypotenuse of a right triangle is 40 centimeters. The legs of the triangle are the same length. How long is each leg of the triangle?

4. A carpenter props a ladder against the wall of a building. The base of the ladder is 10 feet from the wall. The top of the ladder is 24 feet from the ground. How long is the ladder?

The Pythagorean Theorem is very useful. You just have to pay attention to what the problem is asking.



© 2011 Carnegie Learning



Be prepared to share your solutions and methods.